

Anomalous $U(1)'$ and Dark Matter

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Abstract

We have studied the lightest masses in the fermionic sector of an anomalous $U(1)'$ extension of the Minimal Supersymmetric Standard Model (MSSM) inspired by brane constructions. The LSP of this model is an XWIMP (extremely weak interaction particle). We have studied its relic density in the cases in which there is mixing with the neutralinos of the MSSM and in the case in which there is not such mixing. We have showed that this extended model can satisfy the WMAP data. To perform this calculation we have modified the DarkSUSY package.

1. Introduction

There has been much work recently to conceive an intersecting brane model with the gauge and matter content of the Standard Model (SM) of particle physics [1, 2, 3, 4]. One of the features of such models is the presence of extra anomalous gauge $U(1)$'s whose anomaly is cancelled via the Green-Schwarz mechanism. The anomalies of these models are cancelled without assumption on the newly introduced quantum numbers. One model of this type have been introduced and discussed in [6] and [7]. We have studied the compatibility of this model with the WMAP data [8, 9] (see also [10, 11] for related work). In this model we have two new contribution to the neutralinos mass matrix: one coming from the superpartner of the Stückelberg boson (Stückelino) and the other from the superpartner of the gauge boson mediating the extra $U(1)'$ (Primeino). By taking some simplifying and reasonable assumptions on the fermion masses entering the soft supersymmetry breaking lagrangian, the LSP turns out to be mostly a mixture of Stückelino and Primeino. It is also easy to realize that, given the simplifying assumptions mentioned above, the LSP is interacting with the MSSM particles with a coupling suppressed by the inverse of the mass of the extra gauge boson of the theory which must be at least of the order of the TeV for phenomenological reasons. The LSP is then an XWIMP (Extra Weakly Interacting Massive Particle), a class of particles which have already been studied in literature [12, 13]. The cross section of these LSPs is too weak to give the right relic abundance. This is why one has to resort to coannihilations with NLSPs. In our case, the cross section for the annihilations of the LSP with the NLSP and that for the coannihilations of the two species differ for some orders of magnitude. Once again this situation is not new in literature [14, 15, 16] but needs to be treated carefully: the two species will not decouple as far as there will be some MSSM particles to keep them in equilibrium. Moreover these particles must

be relativistic so that their abundance is enough to foster the reaction. There are two cases. First, the anomalous sector of the neutralinos mass matrix can be decoupled from the MSSM sector: this implies that the LSP can be either pure anomalous (Stückelino-Primeino mix) or pure MSSM. This is called the no mixing case and we have studied it in the case in which the LSP is pure anomalous and coannihilates with a pure MSSM NLSP. The second case, called the mixing case, is that in which there are mixing terms between the two sectors of the neutralinos mass matrix. So we can have LSP and NLSP that can be a general mixing of the six gauge eigenstates of the theory. We will show that also in this general case the mixing is constrained and then we will study the case in which the LSP is mostly anomalous with fractions of MSSM eigenstates, while the NLSP is mostly of MSSM origin with small fractions of anomalous eigenstates.

In the subsequent sections we will describe the results for the relic density in both cases. We have obtained that there are regions of the parameter space in which the WMAP data can be satisfied in both cases. These results are obtained using a modified version of the DarkSUSY package which takes into account the couplings of our model.

2. Model

Our model [6] is an extension of the MSSM with an extra $U(1)$. The charges of the matter fields with respect to the symmetry groups are given in table 1.

Gauge invariance imposes constraints that leave independent only three of these charges. In this model we have chosen: Q_Q , Q_L and Q_{H_u} . The anomalies induced by this extension are cancelled by the GS mechanism. Each anomalous triangle diagram is parametrized by a coefficient $b_2^{(a)}$ (entering the lagrangian) with the assignment:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	3	2	1/6	Q_Q
U_i^c	$\bar{3}$	1	-2/3	Q_{U^c}
D_i^c	3	1	1/3	Q_{D^c}
L_i	1	2	-1/2	Q_L
E_i^c	1	1	1	Q_{E^c}
H_u	1	2	1/2	Q_{H_u}
H_d	1	2	-1/2	Q_{H_d}

Table 1: Charge assignment.

$$\mathcal{A}^{(0)} : U(1)' - U(1)' - U(1)' \rightarrow b_2^{(0)} \quad (1)$$

$$\mathcal{A}^{(1)} : U(1)' - U(1)_Y - U(1)_Y \rightarrow b_2^{(1)} \quad (2)$$

$$\mathcal{A}^{(2)} : U(1)' - SU(2) - SU(2) \rightarrow b_2^{(2)} \quad (3)$$

$$\mathcal{A}^{(3)} : U(1)' - SU(3) - SU(3) \rightarrow b_2^{(3)} \quad (4)$$

$$\mathcal{A}^{(4)} : U(1)' - U(1)' - U(1)_Y \rightarrow b_2^{(4)} \quad (5)$$

The mass of the extra boson is parametrized by $M_{V^{(0)}} = 4b_3g_0 \sim 1 \text{ TeV}$, where g_0 is the coupling of the extra $U(1)'$. The terms of the Lagrangian that will contribute to our calculation are [6, 8]:

$$\begin{aligned} \mathcal{L}_{\text{St\"uckelino}} = & \frac{i}{4} \psi_S \sigma^\mu \partial_\mu \bar{\psi}_S - \sqrt{2} b_3 \psi_S \lambda^{(0)} \\ & - \frac{i}{2\sqrt{2}} \sum_{a=0}^2 b_2^{(a)} \text{Tr} \left(\lambda^{(a)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(a)} \right) \psi_S \\ & - \frac{i}{2\sqrt{2}} b_2^{(4)} \left[\frac{1}{2} \lambda^{(1)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(0)} \psi_S + (0 \leftrightarrow 1) \right] + h.c. \end{aligned} \quad (6)$$

The parameters $b_2^{(i)}$ are inversely proportional to $M_{V^{(0)}}$, so they are much smaller than the couplings of the SM. This implies that the primeino and the st\"uckelino are XWIMPS.

The neutral mixing matrix is:

$$\begin{pmatrix} B^\mu \\ W^{3\mu} \\ C^\mu \end{pmatrix} = M \begin{pmatrix} A^\mu \\ Z_0^\mu \\ Z'^\mu \end{pmatrix} \quad (7)$$

Defining $an \equiv g_0 Q_{H_u} \frac{2v^2}{2M_{Z'}^2}$ we have at tree level:

$$M = \begin{pmatrix} c_W & -s_W & s_W \sqrt{g_1^2 + g_2^2} an \\ s_W & c_W & c_W \sqrt{g_1^2 + g_2^2} an \\ 0 & c_W g_2 + s_W g_1 & 1 \end{pmatrix} \quad (8)$$

where c_W is $\cos(\theta_W)$, s_W is $\sin(\theta_W)$. g_1, g_2 are the couplings of the SM electro-weak $SU(2) \times U(1)$ group. The structure of this matrix leaves the electromagnetic sector and the related quantum numbers unchanged with respect to the MSSM ones. We can see from eq. (8) that the case of mixing between the anomalous sector and the MSSM sector is that in which $an \neq 0$, while the case of no mixing

is that in which $an = 0$. From the definition of an it's clear that $an = 0 \Leftrightarrow Q_{H_u} = 0$.

Last, we remember [6, 8] the general form of the neutralinos mass matrix at tree level:

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_S & \sqrt{2} \frac{M_{Z'}}{2} & 0 & 0 & 0 & 0 \\ \dots & M_0 & 0 & 0 & -g_0 v_d Q_{H_u} & g_0 v_u Q_{H_u} \\ \dots & \dots & M_1 & 0 & -\frac{g_1 v_d}{2} & \frac{g_1 v_u}{2} \\ \dots & \dots & \dots & M_2 & \frac{g_2 v_d}{2} & -\frac{g_2 v_u}{2} \\ \dots & \dots & \dots & \dots & 0 & -\mu \\ \dots & \dots & \dots & \dots & \dots & 0 \end{pmatrix}$$

where M_S and M_0 are the soft masses of the st\"uckelino and of the primeino, respectively and v_u, v_d are the vevs (vacuum expectation values) of the Higgs fields. We can see that in case of no mixing ($Q_{H_u} = 0$), there are no terms different from 0 except for the two diagonal boxes that refer to the anomalous and the MSSM sectors.

3. Boltzmann Equation and coannihilations

The relic density of a certain particle is given by the Boltzmann Equation (BE) [17]:

$$\frac{dn}{dt} = -3Hn - \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq}) \quad (9)$$

σ_{ij} is the annihilation cross section:

$$X_i X_j \rightarrow f \bar{f} \quad (10)$$

v_{ij} is the absolute value of relative velocity between i -th and j -th particle, defined by: $v_{ij} = |v_i - v_j|$, n_i is the number of particle per unit of volume of the i -th specie (index 1 refers to LSP), n_i^{eq} is the number of particle per unit of volume of the i -th specie in thermal equilibrium. $n = \sum_i n_i$, H is the Hubble constant and v is the relative velocity of the initial particles.

If the LSP is an XWIMP, it is known [12] that its relic density does not satisfy the WMAP observations [18]. So, in order to achieve the sperimental results, we need coannihilations. This phenomenon occurs when there is at least one particle with mass of the order of the mass of the XWIMP LSP. If the interaction of the coannihilating particles are strong enough they can lower the relic density of the LSP by several orders of magnitude and so we can obtain results in agreement with the sperimental data.

Although a general tractation can be found in [19], for our purposes we are interested only in the cases in which the number N of the coannihilating particles is 2 or 3. In the case $N = 2$ we have the following thermal averaged cross section:

$$\begin{aligned} \langle \sigma_{eff}^{(2)} v \rangle &= \langle \sigma_{11} v \rangle \left(\frac{n_1^{eq}}{n^{eq}} \right)^2 + \\ & 2 \langle \sigma_{12} v \rangle \frac{n_1^{eq} n_2^{eq}}{(n^{eq})^2} + \langle \sigma_{22} v \rangle \left(\frac{n_2^{eq}}{n^{eq}} \right)^2 = \\ & \langle \sigma_{22} v \rangle \frac{\langle \sigma_{11} v \rangle / \langle \sigma_{22} v \rangle + 2 \langle \sigma_{12} v \rangle / \langle \sigma_{22} v \rangle Q + Q^2}{(1 + Q)^2} \end{aligned} \quad (11)$$

where $Q = n_2^{eq}/n_1^{eq}$. The first term in the numerator can be neglected because the Stückelino annihilation cross section is suppressed by a factor $(b_2^{(a)})^4$ with respect to the MSSM neutralino annihilations (see the previous section) and thus $\langle\sigma_{11}v\rangle \ll \langle\sigma_{22}v\rangle$. With the same assumptions of the case $N = 2$ the formula for the case $N = 3$ is:

$$\langle\sigma_{\text{eff}}^{(3)}v\rangle \simeq \frac{\langle\sigma_{22}v\rangle Q_2^2 + 2\langle\sigma_{23}v\rangle Q_2 Q_3 + \langle\sigma_{33}v\rangle Q_3^2}{(1 + Q_2 + Q_3)^2} \quad (12)$$

with

$$Q_i = \frac{n_i^{eq*}}{n_1^{eq}} \quad (13)$$

This expressions show that the factors Q_i can raise/lower the thermal averaged cross section and thus modify the related relic density. However to obtain the result for the relic density it is necessary to numerically solve the BE. To achieve this we have used the DarkSUSY package, adding the interactions introduced by our extended model. In the following section we describe our results.

4. Numerical computations and results

To perform a numerical analysis on the extended model we have modified the DarkSUSY package adding new fields and interactions introduced by the anomalous extension. The free parameters that we use in our numerical simulations are the seven ones used in the MSSM-7 model: the μ mass, the wino soft mass M_2 , the parity-odd Higgs mass M_{A_0} , $tg\beta$, the sfermion mass scale m_{sf} , the two Yukawas a_t and a_b . We add to this set five parameters which define the $U(1)'$ extension: the Stückelino soft mass M_S , the primeino soft mass M_0 , the $U(1)'$ charges Q_{H_u} , Q_Q , Q_L .

4.1. No-mixing case

As we have seen if there is no-mixing we have to impose $Q_{H_u} = 0$. From eq. (9), this implies that the LSP can have either pure anomalous origin or pure MSSM origin. We are interested in the first case. Furthermore, we assume for simplicity that the LSP is a pure Stückelino. Because in this case the anomalous and the MSSM sectors are decoupled, we can choose the model parameters to keep fixed the mass gap between the LSP and the NLSP. So we have studied the behaviour of the model in function of the mass of the LSP for a defined mass gap.

We have studied separately the two cases in which the NLSP is mostly a bino and in which the NLSP is mostly a wino. In the first case the coannihilation involves only the LSP and the NLSP while in the latter case it involves also the lightest chargino, that in the MSSM is almost degenerate in mass with the wino. So they are different situations. The results of the relic density calculations are showed in figures 1 and 2 for the bino NLSP case and in figures 3 and 4 for the wino LSP case.

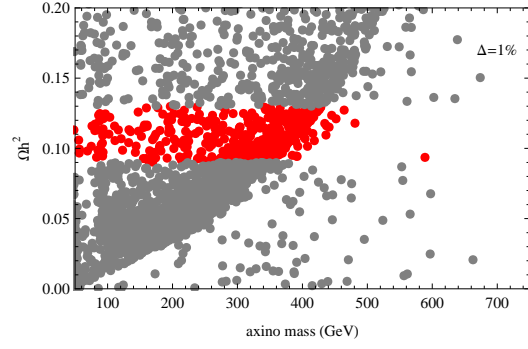


Figure 1: Relic density for a bino-higgsino type NLSP and mass gap 1 %

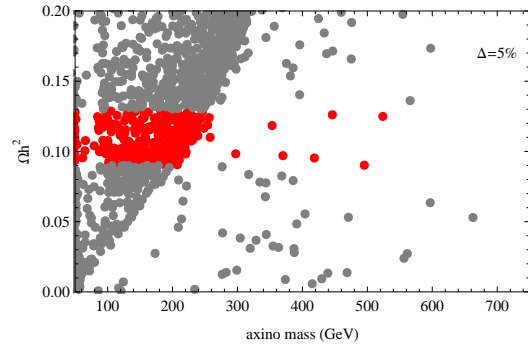


Figure 2: Relic density for a bino-higgsino type NLSP and mass gap 5%

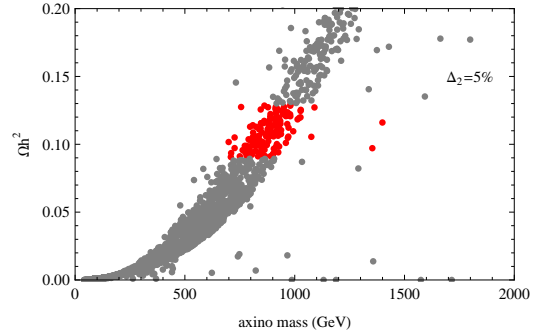


Figure 3: Relic density for a wino type NLSP and mass gap 5%

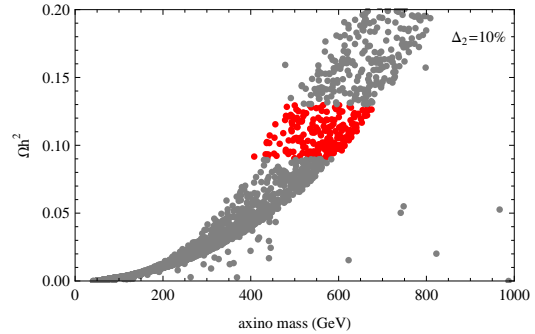


Figure 4: Relic density for a wino type NLSP and mass gap 10 %

In those figures we have shown the most significant results, but we have found model that satisfy WMAP data up to 10% mass gap for the bino NLSP case and up to 20% for the wino NLSP case.

4.2. Mixing case

No mixing implies that $Q_{H_u} \neq 0$, so we can't have control on the mass gap between the LSP and NLSP, because the mass matrix is no more decoupled in two blocks. So we choose to let all parameters unconstrained and therefore to collect data in the mass gap ranges $0 \div 5\%$, $5 \div 10\%$. In each case we have started our study scanning the parameter space in search of the region permitted by the experimental and theoretical constraints, i.e. the region in which we could satisfy the WMAP data with a certain choice of the model parameters. After that we have numerically explored this regions to find sets of parameters that satisfy the WMAP data for the relic density. We have found many suitable combinations for both types of NLSP. So we have chosen some of these successful models and we have computed the relic density keeping constant all but two parameters and plotting the results. We have found regions of the parameter space in which the WMAP data are satisfied for mass gap over 20%, but in the following we will only show results for the regions $0 \div 5\%$ and $5 \div 10\%$, because they are more significant. For simplicity we will refer to these regions as "5% region" and "10% region" respectively.

4.2.1. General results

In this section we want to list some results that are valid for both types of NLSP. First of all, given the constraints on the neutral mixing described in [5], we have obtained that $-1 \lesssim Q_{H_u} \lesssim 1$. This constraint was not appearing in our preliminary analysis in [8]. This implies that also in our general case the mixing between our anomalous LSP and the NLSP is small. We have checked that there are suitable parameter space regions in which the WMAP data are satisfied and we have found that this is true for all possible composition of our anomalous LSP. We have also checked that in each region we have studied there are no divergences or unstable behaviours in our numerical results. We have verified that the relic density is strongly dependent on the LSP and NLSP masses and composition while it is much less dependent on the other variables. Anyway it can be shown that there are cases in which the parameters not related to the LSP or NLSP can play an important role. We will show an example of this case in a forthcoming subsection.

4.2.2. Bino-higgsino NLSP

If the NLSP is mostly a bino-higgsino we have a two particle coannihilation.

We chose two sample models which satisfy the WMAP data [18] with mass gaps 5% and 10%. We study these models to show the dependence of the relic density from the LSP composition and from the mass gap. To obtain these result, we have performed a numerical simulation in which we vary only the stückelino and the primeino soft masses. The results are showed in figure 1, with the conventions:

- Inside the continous lines we have the region in which $(\Omega h^2)_{WMAP} \sim \Omega h^2$
- Inside the thick lines we have the region in which $(\Omega h^2 - 3\sigma)_{WMAP} < \Omega h^2 < (\Omega h^2 + 3\sigma)_{WMAP}$
- Inside the dashed lines we have the region in which $(\Omega h^2 - 5\sigma)_{WMAP} < \Omega h^2 < (\Omega h^2 + 5\sigma)_{WMAP}$
- Inside the dotted lines we have the region in which $(\Omega h^2 - 10\sigma)_{WMAP} < \Omega h^2 < (\Omega h^2 + 10\sigma)_{WMAP}$

Going from the region with a 5% mass gap to that with a 10% mass gap there is a large portion of the parameter space in which the WMAP data cannot be satisfied, while the regions showed in the second and third plot in fig. 1 are similar and thus are mass-gap independent.

4.2.3. Wino-higgsino NLSP

If the NLSP is mostly a wino-higgsino we have a three particle coannihilation, because the lightest wino is almost degenerate in mass with the lightest chargino, so they both contribute to the coannihilations. In this case we perform the same numerical calculation illustrated in the previous subsection. We have extensively studied a sample model with mass gap 10%, showing an example of funnel region, a resonance that occurs when $2 M_{LSP} \sim M_{A_0}$. In our sample model $M_{LSP} \sim 330 \text{ GeV}$, while $M_{A_0} \sim 630 \text{ GeV}$ and this leads to the relic density plot showed in figure 2, with the same conventions used in figure 1. So we can state that also in the case of anomalous LSP we can have a behaviour similar to that of the MSSM, given that the LSP coannihilates with a MSSM NLSP.

5. Conclusion

We have modified the DarkSUSY package in the routines which calculate the cross section of a given supersymmetric particle (contained in the folder `~/src/an`) adding all the new interactions introduced by our anomalous extension of the MSSM. We have also written new subroutines to calculate amplitudes that differ from those already contained in DarkSUSY. We have also modified the routines that generate the supersymmetric model from the inputs, adding the parameters necessary to generate the MiAUSSM [6] and changing the routines that define the model (contained in `~/src/su`) accordingly. Finally we have written a main program that lets the user choose if he wants to perform the relic density calculation in the

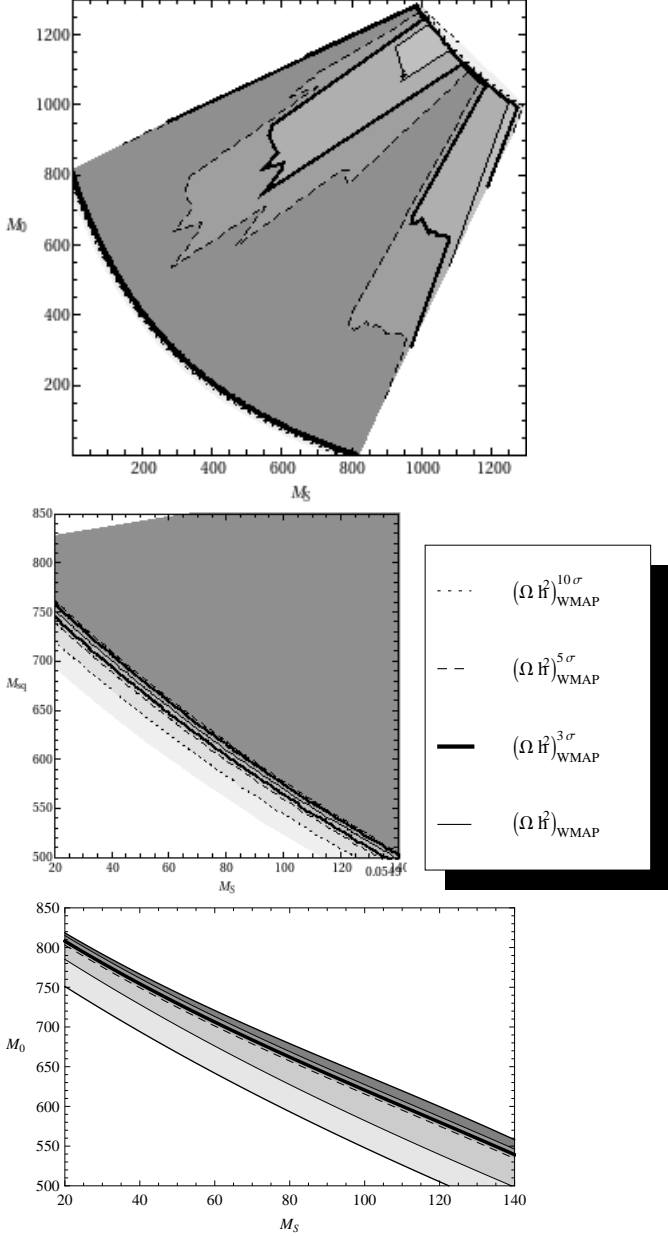


Figure 5: Plot of the relic density of the LSP vs the stükelino and primeino masses. The first plot shows the case of mass gap 5 %. The second plot is a zoom of the first in the region between 20-140 GeV for stükelino mass and 500-850 GeV for primeino mass. The third plot shows the case of mass gap 10 % in the same region of the second plot. All the masses are expressed in GeV

MSSM or in the MiAUSSM. The code of our version of the package is available contacting andrea.mammarella@roma2.infn.it. These modifications have permitted to extensively numerically explore the parameters space for an anomalous extension of the MSSM without restriction on the neutral mixing or on the free parameters of the model. We have verified that our model does not lead to any divergence or instability.

We have studied separately the case in which there is mix-

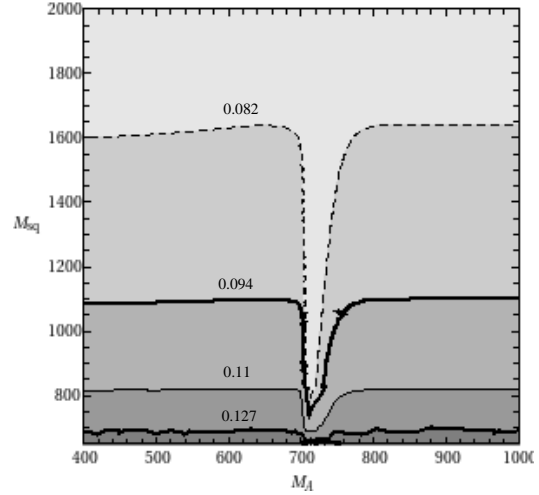


Figure 6: Example of a funnel region for a wino-like NLSP, whose mass gap with the LSP is 10%

ing between the anomalous and the MSSM sectors and the case in which there isn't such mixing.

In the no-mixing case we are able to keep fixed the mass gap between the LSP and the NLSP. We have found that if the NLSP is mostly a bino we can satisfy the WMAP up to 10% mass gap; if the NLSP is mostly a wino we can satisfy the WMAP up to 20% mass gap.

In the mixing case we have we have obtained a constraint on Q_{H_u} . We have also found sizable regions in which we can satisfy the WMAP data for mass gaps which go from 5% to beyond 20%. We have studied some specific sets of parameters for the 5% and 10% mass gap regions, showing that relatively small changes in the mass gap can produce very important changes in the area of the regions which satisfy the experimental constraints.

We have also showed, as an example that we still have the MSSM physics, the presence of a funnel region, analogous to that in the MSSM, in some region of the parameter space.

So we can say that a model with an anomalous LSP can satisfy all the current experimental constraints, can show a phenomenology similar to that expected from a MSSM LSP and can be viable to explain the DM abundance without any arbitrary constraints on its parameters.

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